Calc_Weekly Review 3_0910

1) Evaluate each of the following limits <u>algebraically</u> and discuss the continuity of the function including any points of discontinuity.

a) $\lim_{x\to 3^-} \frac{|x-3|}{x^2-9}$ → $\frac{x-3}{(x-3)(x+3)} \Rightarrow \lim_{x\to 3} \frac{1}{3+3} = -\frac{1}{6}$

b) $\lim_{x\to\infty} \frac{x-2}{x^2+9}$

c) $\lim_{x\to 0} \frac{1-\cos(x)}{x^2} \cdot \frac{1+\cos x}{1+\cos x} \Rightarrow \frac{\sin^2 x}{x^2(1+\cos x)} \Rightarrow \lim_{x\to 0}\frac{\sin x}{x} \cdot \lim_{x\to 0}\frac{\sin x}{x} \cdot \lim_{x\to 0}\frac{1}{1+\cos x}$

$1 \cdot 1 \cdot \frac{1}{2} = \frac{1}{2}$

d) $\lim_{x\to -2} 3x^2 + 5x - 2 = 0$

2) Write the equation of the line tangent to $f(x) = x + \cos x$ at $x = \frac{\pi}{2}$.

4) Use the definition of derivative at a point (p. 100) to find $h'(2)$, if it exists, given that

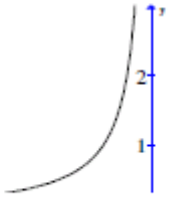
$$h(x) = \begin{cases} 4x^2 + 1, & x \geq 2 \\ 10x - 3, & x < 2 \end{cases}$$

show all work and fully justify your conclusions.

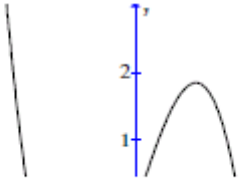
$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

5) Match each graph to the graph of its derivative. Explain your reasoning and sketch both graphs in your answer.

a)



i)

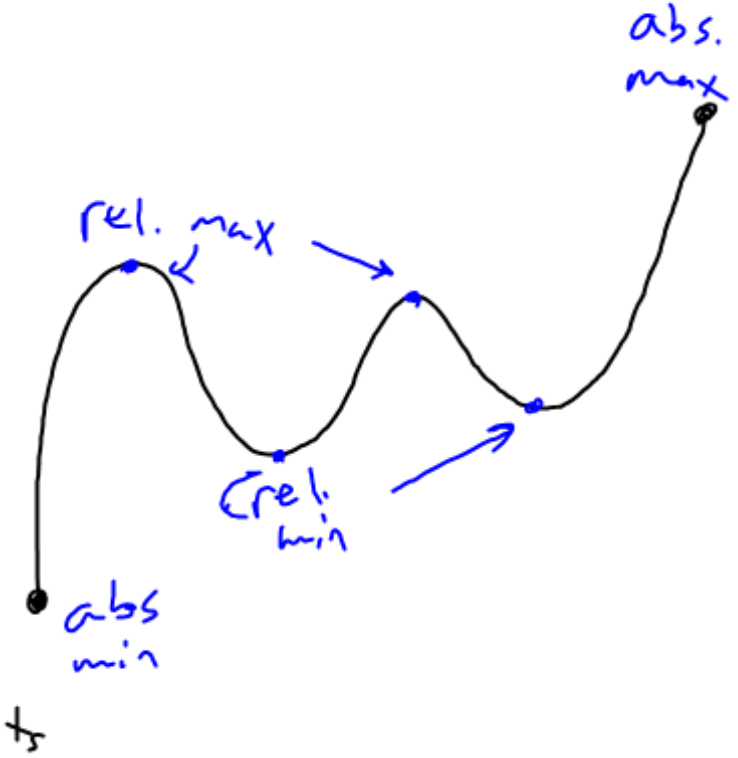


Global (absolute) extreme (max/min) → Smallest biggest ever

closed interval

local (relative) extreme → relative

open interval



Critical Point – $f'(x) = 0$

$f'(x)$ = does it exist

– don't forget endpoints

$y = x^2\sqrt{3-x}$

Find all critical points and label as abs. or relative max/min

Do Algebraically

$y = x^2\sqrt{3-x}$

$2x(\sqrt{3-x}) + x^2 \cdot \frac{1}{2}(3-x)^{-\frac{1}{2}} \cdot -1$

$y' = 2x(\sqrt{3-x}) + \frac{-\frac{x^2}{2}}{\sqrt{3-x}}$

$y' = x\left(\frac{2(\sqrt{3-x}) + \frac{-x}{2}}{\sqrt{3-x}}\right)$

$y' = x\left(\frac{2(\sqrt{3-x}) + \frac{-x}{2}}{\sqrt{3-x}}\right)$

$y' = x\left(\frac{(2\sqrt{3-x})^2 + -x}{2\sqrt{3-x}}\right)$

$\frac{2\sqrt{3-x}}{2\sqrt{3-x}}$

$y' = x\left(\frac{4(3-x) - x}{2\sqrt{3-x}}\right)$

$y' = x\left(\frac{12-4x-x}{2\sqrt{3-x}}\right)$

$y' = x\left(\frac{12-5x}{2\sqrt{3-x}}\right)$

Critical Points

$(0, 0) \rightarrow$ abs min

$\left(\frac{12}{5}, 4.46\right) \rightarrow$ rel. max

$(3, 0) \rightarrow$ abs. min

$x = 0, 3, 2.4$

- Turn in cl. 3 Review
- Sect. 4.1 # 8, 10, 16, 18, 19, 22, 23, 29, 30, 37, 40, 42
- Weekly Review #5